Linear Evaluation of Actual Points Program (LEAPP)

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ABSTRACT

How one intuits a series of points that form one or more lines is a physiological and mathematical problem. To quantify the mathematical portion of this phenomenon, a Linear Evaluation of Actual Points Program (LEAPP) algorithm was written in C++. This program uses the Hough transform to convert Cartesian (x, y) coordinates into Hough space (r, q). Line intersections in Cartesian space are represented by points of intersection or accumulations in Hough space. These intersections are stored in a temporary vote space matrix of size (n = 100, m = 180). The n values span the data set in incremental length units to the maximum Euclidean distance. The m values in the vote space matrix are the compass angles in degrees from 0° (North) to 180° (South). Based on the accumulation of this n by m matrix, physiographic lineations can be represented.

KEYWORDS: Linear Evaluation, Hough Transform.

How one intuits a series of points that form a line or a series of lines is a physiological and mathematical problem. The physiological problem of perception of objects and what constitutes "discernible" and "indiscernible patterns" has been discussed by Dennett (1991). The Hough transform (HT) is a mathematical solution of the problem of detecting straight lines (Hough, 1962). HT has been used for military applications (Casasent and Krishnapuram, 1987; Kiryati and Bruckstein, 1991; Yankowich and Farooq, 1998) and machine vision (Leavers, 1992; Chung and Park, 1994; Ham et al., 1995). No record of its use in evaluating or approximating linear physiographic phenomena has been identified.

Our objective was to develop a computer program to discern alignment of points from physiographic features in two-dimensional space. The Linear Evaluation of Actual Points Program (LEAPP) algorithm was written to determine physiographic lineations using the HT.

THEORY

The basic concept behind HT is that a straight line can be represented by a slope-intercept equation:

$$Y = m * X + b \qquad [1]$$

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where "m" is the slope and "b" is the intercept. Every point on a straight edge falls on a given line in space. Duda and Hart (1972) replaced the slope-intercept formula with the "normal" (perpendicular) representation of a line:

$$\rho = x \cos \theta + y \sin \theta \qquad [2]$$

where "ρ" is the distance from the origin and "θ" is the angle of the line perpendicular to the line of concern. This transforms Cartesian coordinate points (x, y) in real, two-dimensioned space into Hough space coordinates (ρ, θ) that are 90° out of phase with a typical polar coordinate representation. Cartesian coordinates (0, 0) in R^2, are represented by $\rho = 0, 0 < \theta \leq 2\pi$ which is a point in H^2. A line in R^2 from the points (x_1, y_1) and (x_2, y_2) gives rise to the accumulation point (ρ_{12}, θ_{12}) in H^2. Since θ is periodic in 2π radians, for a unique value only π radians (180°) need be evaluated. Therefore, the accumulation point (ρ_{12}, θ_{12}) in H^2 for all ρ values where ρ_{12} and $0 < \theta_{12} \leq 2\pi$ represents the one accumulation point for $0 < \theta \leq \pi$ for the line from (x_1, y_1) to (x_2, y_2) in R^2. To find the alignment of points x_i, y_i in R^2, an n by m matrix called a "vote space matrix" is created. There are n rows (n = 100) representing the distance (r) parameter. There are 180 columns (m = 180) representing θ in one-degree intervals between 0 and π radians. The intersection of lines in R^2 are accumulation points in H^2 and are collected as the cell values of this n by m matrix. The cell values or votes relate to the accumulation points in H^2 or the collinear points in R^2. The discritization constant, m, may be m < 1 or any multiple of 1°. The discritization constant, n, may be n > 0 to a value greater than the geometric distance which spans the data set in R^2. Our program uses n = 100 and associates each individual point with every other point.

VERIFICATION AND DISCUSSION

The ability of LEAPP to discern linear features from physiographic points will be demonstrated from three diverse data sets. A first data set will be the one analyzed by Zartman and Fish (1992). The set of data points, representing the centers of playa lakes in the southwestern portion of Castro County, Texas, are presented in Fig. 1. The LEAPP lines are nearly identical to those presented in Zartman and Fish (1992) for the first four rows of playas below the Running Water Draw. LEAPP lines were quantified from the data points alone, while the regression lines were manually selected. This excellent agreement between the two-presented formats clearly demonstrates the utility of the LEAPP algorithm. The other lines selected by the LEAPP algorithm (Table 2a) also qualitatively demonstrate the utility of this program.

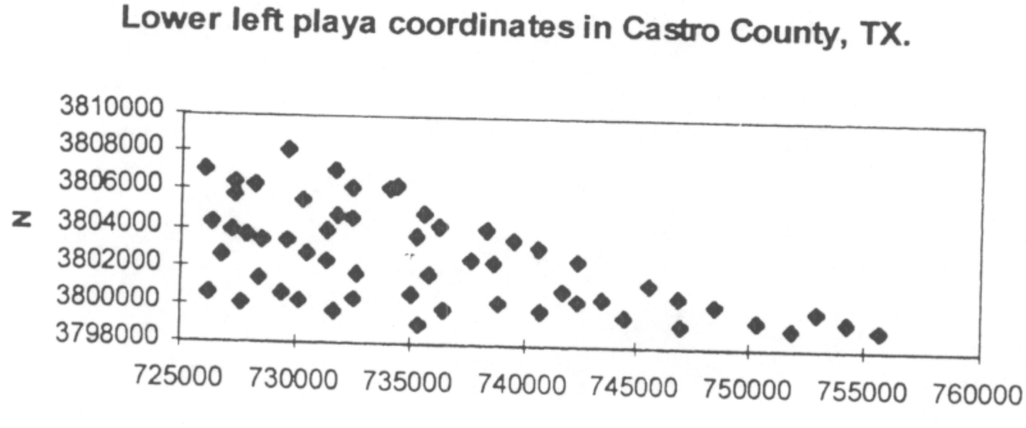

Fig. 1. Coordinates of playas in the lower left corner of Castro County, Texas.

Table 1. Comparison of the LEAPP lines with the Zartman and Fish (1992) line representations. N is Northing and E is Easting in UTM coordinates.

LEAPP line	Zartman and Fish (1992)	Regression value	Row
Y = -0.364 * X + 4.074e + 06	N = 4073140 – 0.364 * E	$R^2 = 0.981$	1
Y = -0.4877 * X + 4.171e + 06	N = 4171546 – 0.500 * E	$R^2 = 0.837$	2
Y = -0.4663 * X + 4.155e + 06	N = 4169465 – 0.499 * E	$R^2 = 0.984$	3
Y = -0.404 * X + 4.103e + 06	N = 4122858 – 0.439 * E	$R^2 = 0.987$	4

Table 2b presents the 13 best angles for the whole data set as determined by LEAPP. These angles are clustered around 112°. These 13 angles account for 29.3% of the angles while arithmetically they should account only for 7.2% (13/180). The non-LEAPP lines were physiologically discriminated against by the presence of the Running Water Draw. Linear regression of the data presented in Fig. 1 was as follows:

North = -0.18446 * East + 3938258 with a nonsignificant $R^2 = 0.33$. This further demonstrates the utility of the LEAPP algorithm for evaluating multiple lines.

Table 2a. The 10 best lines (angle, distance, number of points on the line, and equation) for the lower left corner of Castro County, Texas data set.

Angle	Distance	Number of points	Equation
110	-480.5	7	Y = -0.364 * X + 4.073e + 06
86	-2573	5	Y = 0.06993 * X + 3.749e + 06
93	-3839	5	Y = -0.05241 * X + 3.839e + 06
94	-3710	5	Y = -0.06993 * X + 3.852e + 06
110	-2022	5	Y = -0.364 * X + 4.071e + 06
111	-2003	5	Y = -0.3839 * X + 4.086e + 06
112	-462	5	Y = -0.404 * X + 4.102e + 06
112	-2188	5	Y = -0.404 * X + 4.101e + 06
115	-5134	5	Y = -0.466 * X + 4.143e + 06
116	-5239	5	Y = -0.488 * X + 4.159e + 06

†Negative distances represent values below the center of the data.

Table 2b. The 13 best angles, number of lines (any two points) and percent of total possible lines the Lower left corner of Castro County, Texas data set.

Angle	Number of lines	Percentage of possible
110	51	3.2
112	50	3.1
111	43	2.7
113	39	2.4
86	34	2.1
104	34	2.1
115	34	2.1
109	32	2.0
114	32	2.0
91	30	1.9
96	30	1.9
102	30	1.9
116	30	1.9

A second example of the use of this program is determining the flight pattern of geese (Branta spp.). A photograph of a "vee" of geese (supplied by D. Haukos; U.S. Fish and Wildlife Service) was digitized using the left eyeball of each goose for coordinates (Fig. 2). LEAPP results identify two lines having angles of 77 and 123 degrees, respectively. Linear regression analysis gives one line Y = 0.2188 * X + 5.27 with an R^2 value of 0.105. While the specific angle values of the lines may not be of interest due to distortion during photography, clearly the LEAPP representation of two lines is superior to the single linear regression line.

Goose eyeball coordinates

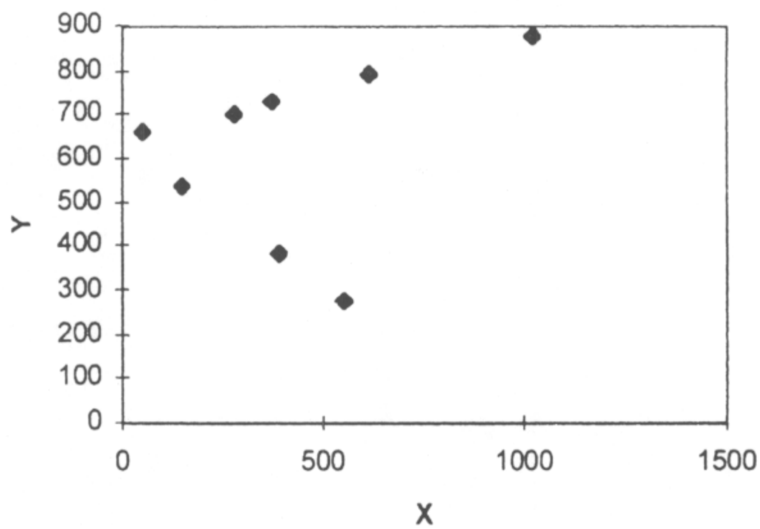

Fig. 2. Relative coordinates for the left eyeball of a flock of geese.

The third data set to be presented are the corners of place squares on a checker board (Fig. 3). LEAPP results presented in Table 3a provide the proper orientation for the 11 best lines. Table 3b presents the best 4 angles selected by LEAPP. These 4 angles account for 30% of the possible angles, but account for the principal axes and diagonal directions. Linear regression analysis of the checkerboard data is Y = 0 * X + 4 with a nonsignificant R^2 value of 0. This data set, again, clearly demonstrated the value of this program compared to a regression analysis of the data. While the human eye perceives the horizontal, vertical, and 45 degree lines, linear regression does not.

CONCLUSIONS

We believe that the utilization of the Hough transform allows us to quantitatively describe linear physiographic features. Random data sets give random values in the vote matrix. Data sets with only one set of points in a line, generate only that line in the vote matrix. With just one line, this algorithm is no better than a least squares procedure. This program is beneficial in detecting multiple lines (Fig. 1). Least squares programs are unable to identify multiple lines and can only give an equation for one line representing the whole data set. By knowing the coordinates of points in real, Cartesian space, transformed data can display the orientation or orientations of these features.

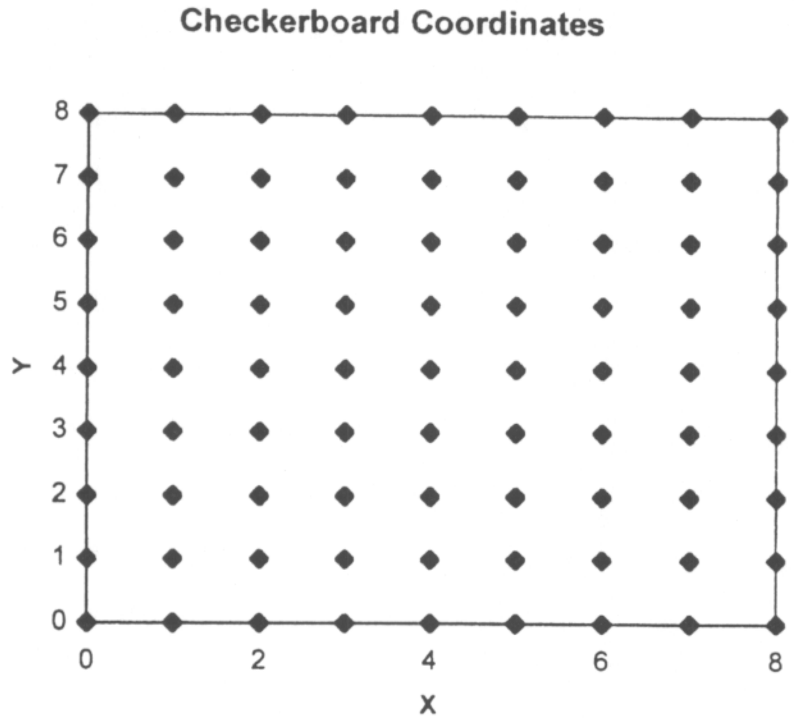

Fig. 3. Coordinates of the corners of place squares on a checkerboard.

Table 3a. The 15 best lines (angle, distance, number of points on the line, and equation) for the corners of place squares on a checkerboard data set.

Angle	Distance	Number of Points	Equation
45	0	9	Y = 1 * X + 5.007e -06
90	4	9	Y = 0 * X + 8
90	3	9	Y = 0 * X + 7
90	2	9	Y = 0 * X + 6
90	1	9	Y = 0 * X + 5
90	0	9	Y = 0 * X + 4
90	-1	9	Y = 0 * X + 3
90	-2	9	Y = 0 * X + 2
90	-3	9	Y = 0 * X + 1
90	-4	9	Y = 0 * X + 2.384e - 07
135	0	9	Y = -0 * X + 8

Table 3b. The 4 best angles, number of lines (any two points) and percent of total possible lines the corners of place squares on a checkerboard data set.

Angle	Number of lines	Percentage of Possible
90	324	10
180	324	10
45	192	5
135	192	5

PROGRAM DESCRIPTION

Program use

The LEAPP algorithm was written in C++ using Windows 97. LEAPP may be run from a 3.5-inch floppy or loaded onto a hard drive and run from that location. For both the floppy and hard disk program, go to the "File" option and chose the "Run" option. Data are entered as columns of numbers in Cartesian coordinates (x, y) or can be read from most commonly utilized spreadsheets. To terminate the data entry, input the coordinates x = 9999 and y = 9999. If the data are read from existing data sets, the terminating coordinates (9999, 9999) need not be entered. The data should be saved as a "Text [Tab delimited]" file with the appropriate ".txt" extension. Once the data are entered or read, a screen menu will display options. Chose the "C" option to compute the actual lines. After the computation is complete, the display option menu will allow the user to display the data or consider other options. Choosing the "F" option will allow the user to input the number of lines to be displayed. Users have the option of selecting the number of lines to be presented with the caveat that the number of lines selected should not exceed the points in the line. The "Q" option will allow the results to be displayed and printed in the directory from which the program was run. The extension ".out" allows the information to be printed from a word processing program.

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